Einstein black holes, free scalars and AdS/CFT correspondence

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Abstract

We investigate AdS/CFT correspondence for two families of Einstein black holes in $d \geq 4$ dimensions, modelling the boundary CFT by a free conformal scalar field and evaluating the boundary two-point function in the bulk geodesic approximation. For the $d \geq 4$ counterpart of the nonrotating BTZ hole and for its $\mathbb{Z}_2$ quotient, the boundary state is thermal in the expected sense, and its stress-energy reflects the properties of the bulk geometry and suggests a novel definition for the mass of the hole. For the generalised Schwarzschild-AdS hole with a flat horizon of topology $\mathbb{R}^{d-2}$, the boundary stress-energy has a thermal form with energy density proportional to the hole ADM mass, but stress-energy corrections from compactified horizon dimensions cannot be consistently included at least for $d = 5$.

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1 Introduction

In the study of AdS/CFT correspondence [1, 2], much attention has focussed on situations where the gravitational side is a black hole solution to an appropriate (super)gravity theory. The dual conformal field theory (CFT) is then expected to contain not only information about the causal structure of the black hole but also information about its quantum properties, in particular the Hawking temperature. While it is difficult to address these questions strictly within the supergravity/CFT pairs in which the evidence for duality is the strongest [1, 3, 4], simplified models that aim to capture aspects of the correspondence have been analysed in various contexts [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. In certain situations a Wilson loop on the conformal boundary can be evaluated semiclassically from extremal world sheets in the bulk, relating critical phenomena in the boundary theory to the classical properties of the bulk black hole [17, 18, 19, 20]. In other situations it has been argued that a boundary two-point function can be evaluated semiclassically from bulk geodesics [13, 14, 27, 28, 29, 30, 31], enabling one to relate the bulk geometry to properties of the boundary state that are recoverable from the two-point function.

The purpose of this paper is to examine the boundary two-point function evaluated semiclassically from bulk geodesics in two families of locally asymptotically anti-de Sitter (AdS) Einstein black holes in $d \geq 4$ dimensions. One aim is to examine how consistently the boundary theory can be modelled by a free conformal scalar field: While a free boundary field has led to consistent results when the bulk is a black hole quotient of AdS_3 [14], it has been argued that for more general black holes a better boundary model may be a high conformal weight field [29]. A second aim is to examine quotients under discrete isometry groups. From the bulk point of view, such a quotient may have a reduced number of globally-defined isometries, and in some cases a time translation isometry may be broken in a way that cannot be detected from just the geometry of an asymptotic region [12, 14, 33, 34, 35]. From the boundary point of view the quotient introduces new geodesics that may contribute to the boundary two-point function, and these geodesics may probe deep bulk regions, in some cases even behind horizons [14]. It becomes thus important to understand in what circumstances such 'long' geodesics can, or indeed must, be included.

We begin with quotients of AdS_d with $d \geq 4$ under a group generated by a single boost-like isometry [36, 37, 38, 39]. These spacetimes are generalisations of the nonrotating $(2+1)$-dimensional Bañados-Teitelboim-Zanelli (BTZ) hole [40, 41], and we refer to them as the higher-dimensional BTZ holes (HD-BTZ holes). They are by construction locally AdS and locally asymptotically AdS. They are black holes, but their exterior region does not have a global timelike Killing vector, the horizon is not stationary, and the infinity is not globally asymptotically AdS. The conformal infinity is connected and has the (conformal) metric $\mathrm{dS}_{d-2} \times S^1$, where the circumference of the S^1 is determined by the magnitude of the boost.

For a free conformal scalar field, the global vacuum on the conformal boundary of AdS_d induces a state on the HD-BTZ boundary $\mathrm{dS}_{d-2} \times S^1$. This state turns out to be

the Euclidean vacuum [42, 43, 44] for each Fourier mode on the circle, and we refer to it as the Euclidean vacuum. We show that a semiclassical evaluation by bulk geodesics reproduces the Euclidean vacuum Green's function, and including the 'long' geodesics is required for this agreement. The vacuum is thermal in the sense of the de Sitter temperature. We compute the difference in the stress-energy tensors in the Euclidean vacua on $\mathrm{dS}_{d-2} \times S^1$ and $\mathrm{dS}_{d-2} \times \mathbb{R}$, showing that this difference decays exponentially in the size of the circle. The stress-energy also suggests a novel definition for the mass of the HD-BTZ hole via AdS/CFT: As the hole is not asymptotically stationary, an Arnowitt-Deser-Misner (ADM) mass is not available.

We then repeat the analysis for a $\mathbb{Z}_2$ quotient of the HD-BTZ hole, analogous to the $\mathbb{RP}^3$ and $\mathbb{RP}^2$ geons constructed from respectively the Schwarzschild hole [33, 34, 35] and the nonrotating BTZ hole [12]. The reduced isometry group now singles out a foliation of the boundary $(\mathrm{dS}_{d-2} \times S^1)/\mathbb{Z}_2$ by spacelike surfaces. We find that the stress-energy contributions from this $\mathbb{Z}_2$ quotient vanish at late and early times in the distinguished foliation but dominate at intermediate times, and in the geodesic approximation the dominant stress-energy contribution at intermediate times comes from the 'long' geodesics created by the $\mathbb{Z}_2$ identification. We also discuss briefly generalisations of the HD-BTZ holes in which the isometry generator is not a pure boost.

Next, we turn to spacetimes obtained from the Schwarzschild-AdS family in $d \geq 4$ dimensions by replacing the round S^{d-2} by flat $\mathbb{R}^{d-2}$ [45, 46, 47, 48, 49]. These generalised Schwarzschild-AdS spacetimes are locally asymptotically AdS but not of locally constant curvature. The conformal boundary consists of two copies of $(d-1)$-dimensional Minkowski space. We evaluate the Green's function on a single boundary component by the geodesic approximation to quadratic order in the coordinate separation, choosing a subtraction procedure compatible with the free scalar field Hadamard form. The stress-energy calculated from the boundary Green's function turns out to be that of a finite temperature free conformal field, the temperature agreeing with the black hole Hawking temperature up to a dimension-dependent numerical factor, and the energy density being proportional to the ADM mass of the hole. At this level, the geodesic method is thus consistent with modelling the boundary theory by a free conformal field even when the spacetime is not locally AdS.

We then replace the flat $\mathbb{R}^{d-2}$ by flat $\mathbb{R}^{d-3} \times S^1$, so that the conformal boundary consists of two copies of $(d-1)$-dimensional Minkowski space with one periodic spatial dimension. Specialising to $d = 5$, we evaluate the leading periodicity correction to the quadratic terms in the boundary Green's function in the limit of large period, finding that this correction does not satisfy the Klein-Gordon equation. Including the 'long' geodesics is thus not consistent with modelling the boundary theory by a free scalar field. We find a similar conclusion for a geon-type variant whose conformal infinity consists of a single copy of four-dimensional Minkowski space with one periodic spatial dimension. In both cases the 'long' geodesics probe the bulk in regions where the local geometry deviates significantly from AdS_d.

We use metric signature $(- + + \cdots)$. The quotient spacetimes of AdS_d are analysed

in section 2 and the generalised Schwarzschild-AdS spacetimes in section 3. Section 4 presents concluding remarks. Certain technical issues are deferred to four appendices.

2 Higher-dimensional BTZ holes and their generalisations

In this section we consider quotient spacetimes of AdS_d with $d \geq 4$. Subsection 2.1 reviews the HD-BTZ construction [36, 37, 38, 39]. The boundary state is analysed in subsections 2.2, 2.3 and 2.4. Subsections 2.5 and 2.6 discuss respectively the further $\mathbb{Z}_2$ quotient and inclusion of rotation.

2.1 HD-BTZ quotients in the bulk and on the boundary

Recall that AdS_d, $d \geq 2$, can be defined as the hyperboloid

$$-\ell^2 = -(X^0)^2 + (X^1)^2 + \cdots + (X^{d-1})^2 - (X^d)^2 \tag{2.1}$$

in $\mathbb{R}^{d-1,2}$ with global coordinates (X^a) and the metric

$$\mathrm{d}s^2 = -(\mathrm{d}X^0)^2 + (\mathrm{d}X^1)^2 + \cdots + (\mathrm{d}X^{d-1})^2 - (\mathrm{d}X^d)^2 \quad . \tag{2.2}$$

The positive parameter ℓ sets the curvature scale. From now on we take $d \geq 4$.

The HD-BTZ hole is the quotient of the region $X^d > |X^{d-1}|$ under the group generated by $\exp(2\pi\lambda_+\xi)$, where ξ is the Killing vector

$$\xi := X^{d-1}\frac{\partial}{\partial X^d} + X^d\frac{\partial}{\partial X^{d-1}} \tag{2.3}$$

and λ_+ is a positive parameter. The subregion that gives the hole exterior is $-(X^0)^2 + (X^1)^2 + \cdots + (X^{d-2})^2 > 0$, $X^d > 0$. A convenient parametrisation of this subregion for our purposes is

$$\begin{aligned}
X^0 &= \ell\sqrt{\rho^2-1}\,\sinh(T) \quad , && (2.4\mathrm{a})\\
X^i &= \ell\sqrt{\rho^2-1}\,\cosh(T)\,\hat{x}^i \quad , && (2.4\mathrm{b})\\
X^{d-1} &= \ell\rho\sinh(\lambda_+\varphi) \quad , && (2.4\mathrm{c})\\
X^d &= \ell\rho\cosh(\lambda_+\varphi) \quad , && (2.4\mathrm{d})
\end{aligned}$$

where $\rho > 1$, $i = 1, \ldots, d-2$ and $\hat{x}$ is a $(d-2)$-dimensional unit vector. The metric in the parametrisation (2.4) reads

$$\mathrm{d}s^2 = \ell^2\left[\frac{\mathrm{d}\rho^2}{\rho^2-1} + (\rho^2-1)\mathrm{d}s^2_{\mathrm{dS}} + \lambda_+^2\rho^2\mathrm{d}\varphi^2\right] \quad , \tag{2.5a}$$

where

$$\mathrm{d}s^2_{\mathrm{dS}} := -\mathrm{d}T^2 + \cosh^2 T\,\mathrm{d}\Omega^2_{d-3} \tag{2.5b}$$

and $\mathrm{d}\Omega^2_{d-3}$ is the metric on unit S^{d-3}. $\mathrm{d}s^2_{\mathrm{dS}}$ (2.5b) is recognised as the global metric on $(d-2)$-dimensional de Sitter space of unit curvature radius. As $\lambda_+\xi = \partial_\varphi$, the HD-BTZ exterior is (2.5) with the identification $(T, \rho, \hat{x}, \varphi) \sim (T, \rho, \hat{x}, \varphi + 2\pi)$. The past and future horizons are located at $\rho = 1$.

The exterior is connected and has spatial topology $S^{d-3} \times S^1$. It has no globally-defined timelike Killing vectors, and the area of the event horizon is increasing in time. Further discussion of the global properties can be found in [36, 37].

The metric on hypersurfaces of constant ρ satisfies

$$\mathrm{d}s^2\big|_{\rho=\mathrm{const}} \sim \ell^2\rho^2\mathrm{d}s^2_{\mathrm{CFT}} \quad , \quad \rho \to \infty \quad , \tag{2.6}$$

where

$$\mathrm{d}s^2_{\mathrm{CFT}} := \mathrm{d}s^2_{\mathrm{dS}} + \lambda_+^2\mathrm{d}\varphi^2 \quad . \tag{2.7}$$

We adopt $\mathrm{d}s^2_{\mathrm{CFT}}$ (2.7) as the representative of the conformal equivalence class of metrics at the infinity. This boundary at the infinity thus the product of $(d-2)$-dimensional de Sitter space of unit curvature radius and a spacelike circle of circumference $2\pi\lambda_+$.

Suppose for the moment that φ were not periodic. In the coordinates $(\tau, \theta, \hat{x})$ defined by

$$\mathrm{e}^{2\lambda_+\varphi} = \frac{\cos\tau - \cos\theta}{\cos\tau + \cos\theta} \quad , \tag{2.8a}$$

$$\tanh T = \frac{\sin\tau}{\sin\theta} \quad , \tag{2.8b}$$

the domain that corresponds to (2.7) would be the diamond $\mathcal{D} := \big\{(\tau, \theta, \hat{x}) \bigm| |\pi/2 - \theta| < \pi/2 - |\tau|\big\}$, and the metric would read

$$\mathrm{d}s^2_{\mathrm{CFT}} = \frac{1}{(\sin^2\theta - \sin^2\tau)}\,\mathrm{d}s^2_{\mathrm{ESU}} \quad , \tag{2.9}$$

where

$$\mathrm{d}s^2_{\mathrm{ESU}} = -\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\Omega^2_{d-3} \quad . \tag{2.10}$$

The metric (2.10) is recognised as the Einstein static universe metric on the conformal boundary of AdS_d [50]. On making φ periodic, equations (2.7)–(2.10) therefore show how the conformal boundary of the HD-BTZ hole is obtained as the quotient of the diamond $\mathcal{D}$ on the conformal boundary of AdS_d. We shall use this observation to characterise a vacuum state in subsection 2.2.

2.2 Euclidean vacuum on the boundary

We consider a free conformal scalar field ϕ in the metric (2.7). We begin with nonperiodic φ and make φ periodic at the end of the subsection.

As the Ricci scalar equals $(d-2)(d-3)$, the wave equation reads

$$-\nabla^{\mu}\nabla_{\mu}\phi + \frac{(d-3)^2}{4}\phi = 0 \quad . \tag{2.11}$$

Separating the φ-dependence as $\mathrm{e}^{im\varphi}$, $m \in \mathbb{R}$, gives on dS_{d-2} a wave equation with the effective mass squared term $\frac{1}{4}(d-3)^2 + (m/\lambda_+)^2$. We denote by Euclidean vacuum the vacuum whose positive frequency mode functions reduce to the dS_{d-2} Euclidean vacuum positive frequency mode functions for each m [42, 43, 44]. A normalised set of such mode functions is

$$\phi_{mnk} := \frac{1}{2\sqrt{2\lambda_+}}\, \mathrm{e}^{im\varphi} (\cosh T)^{(3-d)/2} f_{mn}\bigl(-\tanh T\bigr)\, Y_{nk} \quad , \tag{2.12a}$$

where Y_{nk} are the spherical harmonics on unit S^{d-3} [51, 52], the index n ranges over non-negative integers, the eigenvalue of the scalar Laplacian on S^{d-3} is $-n(n+d-4)$, the index k labels the degeneracy for each n, and

$$f_{mn}(x) := \mathrm{e}^{\frac{1}{2}\pi(i\nu - m/\lambda_+)} \sqrt{\frac{\Gamma\bigl(\nu - im/\lambda_+ + 1\bigr)}{\Gamma\bigl(\nu + im/\lambda_+ + 1\bigr)}} \left[\mathrm{P}_{\nu}^{im/\lambda_+}(x) + \frac{2i}{\pi}\mathrm{Q}_{\nu}^{im/\lambda_+}(x)\right] \quad , \tag{2.12b}$$

where $\mathrm{P}_{\nu}^{im/\lambda_+}$ and $\mathrm{Q}_{\nu}^{im/\lambda_+}$ are the associated Legendre functions on the cut [53] and $\nu = n + \frac{1}{2}(d-5)$. The linear combination of $\mathrm{P}_{\nu}^{im/\lambda_+}$ and $\mathrm{Q}_{\nu}^{im/\lambda_+}$ in (2.12b) is determined by the analytic continuation properties [44, 54], as can be verified using the formulas in [55], p. 168.

By (2.9), the functions $\psi_{mnk} := (\sin^2\theta - \sin^2\tau)^{(3-d)/4}\phi_{mnk}$ solve the conformal scalar field equation in the diamond $\mathcal{D}$ in the Einstein static universe (2.10). On analytic continuation outside $\mathcal{D}$, it can be verified ([55], p. 168) that ψ_{mnk} are bounded in the lower half-plane in complexified τ and hence purely positive frequency with respect to the Killing vector ∂_τ. This means that our Euclidean vacuum is the vacuum induced from the global vacuum on the conformal boundary of AdS_d.

To evaluate the Euclidean vacuum Green's function, we recall that the Euclidean vacuum Green's function $G_{\mathrm{dS}}^{(m)}$ on dS_{d-2} for a scalar field with mass squared $\frac{1}{4}(d-3)^2 + (m/\lambda_+)^2$ reads [44]

$$G_{\mathrm{dS}}^{(m)}(T,\hat{x};T',\hat{x}') = \frac{\Gamma\bigl(\frac{d-3}{2} + i\frac{m}{\lambda_+}\bigr)\Gamma\bigl(\frac{d-3}{2} - i\frac{m}{\lambda_+}\bigr)}{2(2\pi)^{(d-2)/2}} (1-Z^2)^{(4-d)/4}\, \mathrm{P}^{(4-d)/2}_{-\frac{1}{2}+im/\lambda_+}(-Z) \quad , \tag{2.13}$$

where the coordinates on dS_{d-2} are as in the first two terms in (2.7),

$$Z(T,\hat{x};T',\hat{x}') := \cosh(T)\cosh(T')(\hat{x}\cdot\hat{x}') - \sinh(T)\sinh(T') \quad , \tag{2.14}$$

$\hat{x}\cdot\hat{x}' := \sum_i x^i x'^i$, and we have written the hypergeometric function of [44] in terms of the associated Legendre function on the cut [53]. Formula (2.13) assumes $-1 < Z < 1$, and

an appropriate analytic continuation gives $G_{\mathrm{dS}}^{(m)}$ for other values of Z [44]. By Fourier analysis in φ, the Green's function in the metric (2.7) thus reads

$$\begin{aligned} G(T,\hat{x},\varphi;T',\hat{x}',\varphi') &= \frac{1}{2\pi\lambda_+}\int_{-\infty}^{\infty} \mathrm{d}m\, \mathrm{e}^{im(\varphi-\varphi')}\, G_{\mathrm{dS}}^{(m)}(T,\hat{x};T',\hat{x}') \\ &= \frac{\Gamma\left(\frac{d-3}{2}\right)}{2(2\pi)^{(d-1)/2}} \times \frac{1}{\left[\cosh(\lambda_+\Delta\varphi) - Z(T,\hat{x};T',\hat{x}')\right]^{(d-3)/2}} \quad , \end{aligned} \tag{2.15}$$

where $\Delta\varphi := \varphi - \varphi'$ and we have evaluated the integral over m using 7.217 in [53]. The denominator in (2.15) is positive for $Z < \cosh(\lambda_+\Delta\varphi)$, and an appropriate analytic continuation gives $G(T,\hat{x},\varphi;T',\hat{x}',\varphi')$ for other values of Z. It can be readily verified that (2.15) satisfies the wave equation (2.11). Note that conformally transforming (2.15) to the Einstein static universe by (2.7)–(2.10) gives the Einstein static universe Green's function in the vacuum that is positive frequency with respect to ∂_τ.

The Green's function (2.15) is periodic in T with period $2\pi i$. This can be understood as a consequence of the periodicity of the Euclidean vacuum Green's function on dS_{d-2} [56]. By the contour deformation argument in [57], the response function of an inertial monopole particle detector at constant φ satisfies the Kubo-Martin-Schwinger (KMS) condition at temperature $1/(2\pi)$, and our Euclidean vacuum is thus thermal in this sense. The contour deformation argument does however not generalise to an inertial particle detector that has velocity in the φ direction, and we have found no reason to expect the response of such a detector to be thermal.

Finally, turn to the boundary of the HD-BTZ hole, where φ has period 2π. The separation constant m in the modes ϕ_{mnk} (2.12) then becomes an integer. The complex analytic properties of the modes ψ_{mnk} on the Einstein static universe remain unchanged, and the Euclidean vacuum is still that induced from the global vacuum on the conformal boundary of AdS_d. The Green's function is obtained from (2.15) by the method of images, with the result

$$G(x;x') = \frac{\Gamma\left(\frac{d-3}{2}\right)}{2(2\pi)^{(d-1)/2}} \times \sum_{k\in\mathbb{Z}} \frac{1}{\left\{\cosh\left[\lambda_+(\Delta\varphi + 2\pi k)\right] - Z(T,\hat{x};T',\hat{x}')\right\}^{(d-3)/2}} \quad . \tag{2.16}$$

The contour deformation argument of [57] shows again that the response function of an inertial monopole particle detector at constant φ satisfies the KMS condition at temperature $1/(2\pi)$.

2.3 Boundary Green's function from bulk geodesics

In this subsection we show that the bulk geodesic approximation of [13] reproduces the Euclidean vacuum Green's function on the conformal boundary of the HD-BTZ hole.

Recall that in the geodesic method one first assumes for an appropriate bulk Green's function the estimate

$$G_{\mathrm{bulk}}(y,y) \sim P \exp[-\mu L(y,y')] \quad , \tag{2.17}$$

where $L(y, y')$ is the length of a (spacelike) geodesic connecting the bulk points y and y', μ is a constant and P is a prefactor that is slowly varying in some suitable sense. When y and y' tend to two spacelike-separated points on the conformal boundary, $L(y, y')$ diverges, but a finite remainder may be obtained via multiplicative renormalisation of (2.17) by a function of the hypersurfaces through which y and y' approach the conformal boundary. The aim is to interpret this remainder as a boundary Green's function of a conformal scalar field whose conformal weight depends on μ.

Consider now AdS_d in the parametrisation (2.4), take $y = (T, \rho, \hat{x}, \varphi)$, $y' = (T', \rho, \hat{x}', \varphi')$, and let $\rho \to \infty$. Writing $D := -(\Delta X^0)^2 + (\Delta X^1)^2 + \cdots + (\Delta X^{d-1})^2 - (\Delta X^d)^2$, a direct computation gives

$$\frac{D(y, y')}{2\ell^2} = -1 + \rho^2 \left[\cosh(\lambda_+ \Delta\varphi) - \left(1 - \rho^{-2}\right) Z(T, \hat{x}; T', \hat{x}')\right] \quad , \tag{2.18}$$

where Z was defined in (2.14). From the symmetries of AdS_d it follows that the relation between D and the geodesic distance L is

$$\sinh^2\left(\frac{L}{2\ell}\right) = \frac{D}{4\ell^2} \quad . \tag{2.19}$$

For $\mu > 0$, (2.18) and (2.19) imply

$$\mathrm{e}^{-\mu L} = \left(2\rho^2\right)^{-\mu\ell} \times \frac{1}{\left[\cosh(\lambda_+ \Delta\varphi) - Z(T, \hat{x}; T', \hat{x}')\right]^{\mu\ell}} \times \left[1 + \mathcal{O}\left(\rho^{-2}\right)\right] \quad . \tag{2.20}$$

With the choice $\mu\ell = \frac{1}{2}(d-3)$, (2.20) thus gives

$$\frac{\Gamma\left(\frac{d-3}{2}\right)}{4\pi^{(d-1)/2}} \times \rho^{2\mu\ell}\, \mathrm{e}^{-\mu L} \xrightarrow[\rho\to\infty]{} G(T, \hat{x}, \varphi; T', \hat{x}', \varphi') \quad , \tag{2.21}$$

with $G(T, \hat{x}, \varphi; T', \hat{x}', \varphi')$ given by (2.15). This shows that the geodesic approximation with $\mu\ell = \frac{1}{2}(d-3)$ reproduces the Euclidean vacuum Green's function on $\mathrm{dS}_{d-2} \times \mathbb{R}$. Taking the periodic sum in φ reproduces the Euclidean vacuum Green's function on the boundary of the HD-BTZ hole.

The geodesics involved in the result (2.21) are real and spacelike for $\cosh(\lambda_+\Delta\varphi) - Z(T, \hat{x}; T', \hat{x}') > 0$, while for other values the geodesic approximation is understood in the sense of analytic continuation. As geodesics in de Sitter space have $Z \geq -1$ [58], it follows from the ds^2_{dS} term in (2.5a) that the real spacelike geodesics for which $Z < -1$ have to pass inside the HD-BTZ horizon.

2.4 Boundary stress-energy

In AdS/CFT correspondence, one expects boundary stress-energy to be related to the bulk mass, assuming a bulk mass can be independently defined. We now examine this issue for the HD-BTZ hole.

Consider the boundary. The symmetries of the metric (2.7) and the Green's function (2.16) imply that the Euclidean vacuum stress-energy expectation value takes the form $\langle T_{\mu\nu}\rangle = a(g_{\rm dS})_{\mu\nu} + b\big[g_{\mu\nu} - (g_{\rm dS})_{\mu\nu}\big]$, where a and b are constants and $(g_{\rm dS})_{\mu\nu}$ is the de Sitter metric (2.5b). To determine a and b, one starts from the classical expression for $T_{\mu\nu}$, which in our conventions reads [59]

$$
\begin{aligned}
T_{\mu\nu} = {} & \frac{d-1}{2(d-2)}\phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2(d-2)} g_{\mu\nu} g^{\rho\sigma}\phi_{,\rho}\,\phi_{,\sigma} \\
& - \frac{d-3}{2(d-2)}\phi_{;\mu\nu}\phi + \frac{(d-3)^2}{8(d-2)}\big[-g_{\mu\nu} + 2(g_{\rm dS})_{\mu\nu}\big]\phi^2 \quad . \qquad (2.22)
\end{aligned}
$$

One point-splits (2.22), reinterprets it in terms of the Green's function, and finally takes the coincidence limit after subtracting the divergent geometric part [59].

The main computational effort in this point-splitting would be in the divergent geometric part, which becomes increasingly complicated with increasing dimension. As our principal interest is in the λ_+-dependence of $\langle T_{\mu\nu}\rangle$, we only evaluate $\Delta T_{\mu\nu}$, the difference of $\langle T_{\mu\nu}\rangle$ between the spacetimes with periodic and nonperiodic φ. $\Delta T_{\mu\nu}$ is the contribution from the $k \neq 0$ terms in (2.16) and requires no renormalisation. A straightforward computation shows that the only nonvanishing components are

$$
\Delta T^T_T = \frac{\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)(2\pi)^{(d-1)/2}} \sum_{k\neq 0} \frac{(d-3)\cosh(2\pi k\lambda_+) + (d-1)}{\big[\cosh(2\pi k\lambda_+)\big]^{(d-1)/2}} \quad , \qquad (2.23a)
$$

$$
\Delta T^i_j = \Delta T^T_T\,\delta^i_j \quad , \qquad (2.23b)
$$

$$
\Delta T^\varphi_\varphi = -(d-2)\Delta T^T_T \quad , \qquad (2.23c)
$$

where the indices in (2.23b) are on S^{d-3}. Note that $\Delta T_{\mu\nu}$ is traceless.

Now, the energy density seen by an inertial observer at constant φ in the metric (2.7) is $E = E_0 - \Delta T^T_T$, where E_0 is the λ_+-independent contribution from the $k = 0$ term in (2.16) after renormalisation. In the limit of large λ_+, the dominant terms in (2.23a) are those with $k = \pm 1$, and we find

$$
E \sim E_0 - \frac{(d-3)\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)\pi^{(d-1)/2}}\, \mathrm{e}^{-(d-3)\pi\lambda_+} \quad . \qquad (2.24)
$$

For a large (and hence presumably classical) HD-BTZ hole, $\lambda_+ \gg 1$, AdS/CFT correspondence thus suggests associating with the hole the CFT mass

$$
M_{\rm CFT} = A\left(1 - \mathrm{e}^{-(d-3)\pi\lambda_+}\right) \quad , \qquad (2.25)
$$

where A is some positive constant and we have normalised the zero of $M_{\rm CFT}$ to $\lambda_+ = 0$.

Consider then the bulk. As the neighbourhood of the infinity is not asymptotically stationary, an ADM mass is not defined. For $d = 5$, a conserved charge proportional to λ_+^2 was identified by embedding the bulk in Chern-Simons supergravity [38, 39]. In

comparison, the $d = 3$ ADM mass is proportional to λ_+^2 [40, 41]. We note that the relation (2.25) between $M_{\rm CFT}$ and λ_+ resembles the relation between the ADM energy H and C-energy c for cylindrical gravitational waves in four dimensions,

$$H = \frac{1}{4G}\left(1 - \mathrm{e}^{-4Gc}\right) \quad , \tag{2.26}$$

where G is Newton's constant [60]. It would be interesting to understand whether this resemblance is more than a coincidence.

An elementary analysis shows that the $k \neq 0$ coincidence limit geodesics are contained in the HD-BTZ exterior and pass asymptotically close to the horizon as $\lambda_+|k| \to \infty$. From the geodesic approximation viewpoint, $\Delta T_{\mu\nu}$ does therefore not probe the geometry behind the horizons but the result (2.24) probes the near-horizon region of the exterior.

2.5 $\mathbb{Z}_2$ quotient

We next consider the quotient of the region $X^d > |X^{d-1}|$ in AdS_d under the group generated by the isometry $J := \exp(\pi\lambda_+\xi) \circ J_0$, where

$$J_0 : \left(X^0, X^1, \cdots, X^{d-2}, X^{d-1}, X^d\right) \mapsto \left(X^0, -X^1, \cdots, -X^{d-2}, X^{d-1}, X^d\right) \quad . \tag{2.27}$$

As $J^2 = \exp(2\pi\lambda_+\xi)$, this spacetime is a $\mathbb{Z}_2$ quotient the HD-BTZ hole. The construction resembles that of the $\mathbb{RP}^3$ and $\mathbb{RP}^2$ geons as $\mathbb{Z}_2$ quotients of respectively Kruskal [33, 34, 35] and the nonrotating BTZ hole [12], but while the $\mathbb{RP}^3$ and $\mathbb{RP}^2$ geon quotients identify two disconnected exterior regions, the exterior of the HD-BTZ hole is already connected, and the $\mathbb{Z}_2$ identification in the exterior coordinates (2.5) reads

$$(T, \rho, x^i, \varphi) \sim (T, \rho, -x^i, \varphi + \pi) \quad . \tag{2.28}$$

The metric on the conformal boundary is given by (2.5b) and (2.7) with the identification

$$(T, x^i, \varphi) \sim (T, -x^i, \varphi + \pi) \quad . \tag{2.29}$$

Note that among the continuous de Sitter isometries in (2.7), (2.29) preserves only the rotations on S^{d-3}. The boundary has thus a distinguished spacelike foliation by the constant T hypersurfaces. Similarly, the bulk exterior has a distinguished spacelike foliation by the constant T hypersurfaces in (2.5).

The Euclidean vacuum on the conformal boundary of the HD-BTZ hole induces a Euclidean vacuum on the conformal boundary of the $\mathbb{Z}_2$ quotient. By the method of images, the Green's function is the sum of (2.16) and the additional term

$$\begin{aligned} &\Delta_g G(x;x') \\ &\quad := \frac{\Gamma\left(\frac{d-3}{2}\right)}{2(2\pi)^{(d-1)/2}} \times \sum_{k\in\mathbb{Z}} \frac{1}{\left(\cosh\left\{\lambda_+[\Delta\varphi + (2k+1)\pi]\right\} - Z(T,\hat{x};T',-\hat{x}')\right)^{(d-3)/2}} \quad , \end{aligned} \tag{2.30}$$

and it follows from subsection 2.3 that the term (2.30) is reproduced by the bulk geodesic approximation.

The contribution from $\Delta_g G(x;x')$ (2.30) to the stress-energy tensor requires no renormalisation. In the notation of (2.23), a direct computation yields

$$\Delta_g T_T^T = \frac{(d-3)\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)(2\pi)^{(d-1)/2}} \sum_{k\in\mathbb{Z}} \frac{C_k+1}{(C_k-Z)^{(d-1)/2}} \quad , \tag{2.31a}$$

$$\Delta_g T_j^i = \delta_j^i \, \frac{\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)(2\pi)^{(d-1)/2}} \sum_{k\in\mathbb{Z}} \frac{(C_k+1)\left[d(C_k-1)+2(Z-C_k)\right]}{(C_k-Z)^{(d+1)/2}} \quad , \tag{2.31b}$$

$$\Delta_g T_\varphi^\varphi = -\frac{(d-3)\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)(2\pi)^{(d-1)/2}} \sum_{k\in\mathbb{Z}} \frac{dC_k^2-1+(C_k+1)(Z-C_k)}{(C_k-Z)^{(d+1)/2}} \quad , \tag{2.31c}$$

where $C_k := \cosh[(2k+1)\pi\lambda_+]$ and $Z = -\cosh(2T)$. Note that $\Delta_g T_{\mu\nu}$ is traceless.

As the boundary is not globally de Sitter invariant, there is no reason to expect $\Delta_g T_{\mu\nu}$ to be locally de Sitter invariant, and the T-dependence in (2.31) shows that it indeed is not. We now show that the relative magnitude of $\Delta T_{\mu\nu}$ and $\Delta_g T_{\mu\nu}$ depends on both λ_+ and T.

Consider first the limit of large λ_+ with fixed T. The leading terms in $\Delta_g T_{\mu\nu}$ (2.31) are those with $k=0$ and -1, proportional to $\exp\left[-\frac{1}{2}(d-3)\pi\lambda_+\right]$, and the T-dependence only appears in a subleading order. In comparison, the leading terms in $\Delta T_{\mu\nu}$ (2.23) are proportional to $\exp\left[-(d-3)\pi\lambda_+\right]$. Hence $\Delta_g T_{\mu\nu}$ dominates $\Delta T_{\mu\nu}$.

Consider then the limit of large $|T|$ with fixed λ_+. In (2.31), we first arrange the sums in the form $\sum_{k=0}^{\infty}(C_k-Z)^{-p}$ with $p>0$ and use the results in appendix A to identify the leading sums and replace C_k by $\frac{1}{2}\mathrm{e}^{\pi\lambda_+}\mathrm{e}^{2\pi k\lambda_+}$. We then rearrange the leading sums into sums of the form

$$\sum_{k=0}^{\infty} \frac{\frac{1}{2}(-Z)^{-1}\mathrm{e}^{\pi\lambda_+}\mathrm{e}^{2\pi k\lambda_+}}{\left[\frac{1}{2}(-Z)^{-1}\mathrm{e}^{\pi\lambda_+}\mathrm{e}^{2\pi k\lambda_+}+1\right]^p} \quad , \quad p>1 \quad . \tag{2.32}$$

In the sums (2.32), we next include also negative integer values of k: This introduces an error of order $\mathcal{O}\left(Z^{-1}\right)$, but as the new sums are periodic in $\ln(-Z)$, the error is sub-leading. Finally, replacing Z by $-\frac{1}{2}\mathrm{e}^{2|T|}$, we find the asymptotic large $|T|$ behaviour

$$\Delta_g T_T^T \sim \frac{(d-3)\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)\pi^{(d-1)/2}} \, \mathrm{e}^{-(d-3)|T|} \, f_{(d-1)/2} \quad , \tag{2.33a}$$

$$\Delta_g T_j^i \sim \delta_j^i \, \frac{\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)\pi^{(d-1)/2}} \, \mathrm{e}^{-(d-3)|T|} \left[(d-3)f_{(d-1)/2}-(d-1)f_{(d+1)/2}\right] \quad , \tag{2.33b}$$

$$\Delta_g T_\varphi^\varphi \sim -\frac{(d-3)\Gamma\left(\frac{d-1}{2}\right)}{4(d-2)\pi^{(d-1)/2}} \, \mathrm{e}^{-(d-3)|T|} \left[(d-2)f_{(d-1)/2}-(d-1)f_{(d+1)/2}\right] \quad , \tag{2.33c}$$

where

$$f_p := \sum_{k\in\mathbb{Z}} \frac{\mathrm{e}^{(2k+1)\pi\lambda_+ - 2|T|}}{\left[\mathrm{e}^{(2k+1)\pi\lambda_+ - 2|T|} + 1\right]^p} \quad , \quad p > 1 \quad . \tag{2.34}$$

Note that f_p is periodic in $|T|$, with period $\pi\lambda_+$, and hence bounded in $|T|$.[1] Equations (2.33) thus show that $\Delta_g T_{\mu\nu}$ decays exponentially as $|T| \to \infty$.

For given $\lambda_+ \gg 1$, these results imply that $\Delta_g T_{\mu\nu}$ dominates $\Delta T_{\mu\nu}$ for some finite interval in T but decays exponentially as $|T| \to \infty$. This decay could be expected from similar results in geon-type versions of Rindler space [62, 63] and in the $\mathbb{RP}^3$ version of de Sitter space [64]. Note that the dominance of $\Delta_g T_{\mu\nu}$ does not contradict the proposal (2.25) for the mass of the HD-BTZ hole, as the infinity neighbourhoods in the HD-BTZ hole and the $\mathbb{Z}_2$ quotient are not globally isometric. Whether $\Delta_g T_{TT} + \Delta T_{TT}$ might provide a reasonable mass for the $\mathbb{Z}_2$ quotient is not clear, but the T-dependence of such a mass would not contradict any bulk symmetries.

Finally, note that all the new $T \neq 0$ coincidence limit geodesics have $Z < -1$ and hence pass inside the horizon. From the geodesic approximation viewpoint this means that $\Delta_g T_{\mu\nu}$ arises entirely from geodesics that probe the geometry behind the horizon.

2.6 Adding rotation to the HD-BTZ hole

The HD-BTZ hole is a direct generalisation of the nonrotating BTZ hole to $d \geq 4$. Generalising the rotating BTZ hole in a similar way produces a spacetime that is not a black hole [37], but part of the conformal boundary of this spacetime is still obtained from the metric (2.7) by an appropriate identification, and the methods of subsections 2.2 and 2.3 define on this part of the boundary a conformal scalar field state. We now discuss briefly the stress-energy of this state in view of AdS/CFT correspondence.

In the bulk, the isometry group is now generated by $\exp\bigl(2\pi\lambda_+\xi + \lambda_-\eta\bigr)$, where ξ is as in (2.3),

$$\eta := X^0 \frac{\partial}{\partial X^{d-2}} + X^{d-2}\frac{\partial}{\partial X^0} \quad , \tag{2.35}$$

and the parameters λ_+ and λ_- satisfy $0 < \lambda_- < 2\pi\lambda_+$. The region of interest on the conformal boundary without identifications is (2.7) with $-\infty < \varphi < \infty$, and the group acting on it is generated by $\exp\bigl(2\pi\partial_\varphi + \lambda_-\tilde\eta\bigr)$, where $\tilde\eta$ is induced by η: $\tilde\eta$ is a boost-like Killing vector on $\mathrm{d}s^2_{\mathrm{dS}}$ in (2.7), normalised so that $\tilde\eta^\mu\tilde\eta_\mu \geq -1$. Note that $2\pi\partial_\varphi + \lambda_-\tilde\eta$ is spacelike. In a static coordinate patch $(\tilde t, \tilde r, \tilde x^i, \varphi)$ adapted to $\tilde\eta$, the metric (2.7) reads

$$\mathrm{d}s^2_{\mathrm{CFT}} = -(1-\tilde r^2)\mathrm{d}\tilde t^2 + \left(1-\tilde r^2\right)^{-1}\mathrm{d}\tilde r^2 + \tilde r^2 \mathrm{d}\tilde\Omega^2_{d-4} + \lambda_+^2 \mathrm{d}\varphi^2 \quad , \tag{2.36}$$

where $0 \leq \tilde r < 1$, $\tilde\eta = \partial_{\tilde t}$, and the identification is

$$(\tilde t, \tilde r, \tilde x^i, \varphi) \sim (\tilde t + \lambda_-, \tilde r, \tilde x^i, \varphi + 2\pi) \quad , \tag{2.37}$$

[1] Viewed as a function of complexified $|T|$, f_p has also period πi and is hence an elliptic function. For $p = 2$, it can be expressed in terms of the Weierstrass elliptic function ([61], pp. 45–47).

where $\tilde{x}$ is the $(d-3)$-dimensional unit vector coordinatising S^{d-4}.

The Green's function is constructed from (2.15) by the method of images, and the contribution to the boundary stress-energy tensor from the identifications can be computed as in subsection 2.4. Working in the static coordinates (2.36), the differentiations can be performed with the help of the formula

$$Z(x;x') = \sqrt{(1-\tilde{r}^2)(1-\tilde{r}'^2)\cosh(\Delta\tilde{t}) + \tilde{r}\tilde{r}'\sum_{i=1}^{d-3}\tilde{x}^i\tilde{x}'^i} \quad . \tag{2.38}$$

In the limit of large λ_+ with fixed λ_-/λ_+, the leading asymptotic behaviour is

$$\Delta T^{\tilde{t}}_{\tilde{t}} \sim F\left[(d-3) + (d-1-2\tilde{r}^2)q + (d-2)(d-3)(1-\tilde{r}^2)q^2\right] \quad , \tag{2.39a}$$

$$\Delta T^{\tilde{r}}_{\tilde{r}} \sim F(d-3)\left[1 + \tilde{r}^2 q - (1-\tilde{r}^2)q^2\right] \quad , \tag{2.39b}$$

$$\Delta T^{\tilde{i}}_{\tilde{j}} \sim F\left[(d-3) - 2\tilde{r}^2 q - (d-3)(1-\tilde{r}^2)q^2\right] \quad , \tag{2.39c}$$

$$\Delta T^{\varphi}_{\varphi} \sim -F\left[(d-2)(d-3) + \big((d-3)(1-\tilde{r}^2)+2\big)q + (d-3)(1-\tilde{r}^2)q^2\right] \quad , \tag{2.39d}$$

$$\Delta T^{\tilde{t}}_{\varphi} \sim -F\lambda_+ d(d-2)q \quad , \tag{2.39e}$$

where

$$F := \frac{\Gamma\left(\frac{d-1}{2}\right)\mathrm{e}^{-(d-3)\pi\lambda_+}}{4(d-2)\pi^{(d-1)/2}\left[1-(1-\tilde{r}^2)q\right]^{(d+1)/2}} \quad , \tag{2.40a}$$

$$q := \mathrm{e}^{\lambda_- - 2\pi\lambda_+} \quad . \tag{2.40b}$$

Note that $\Delta T^{\tilde{t}}_{\varphi}$ is nonvanishing, but at large λ_+ with fixed λ_-/λ_+ it is exponentially suppressed compared with the diagonal components.

Now, AdS/CFT correspondence suggests seeking in the boundary stress-energy (2.39) evidence of rotation in the bulk spacetime, and possibly even a definition of the angular momentum of the bulk spacetime. While the nonvanishing value of $\Delta T^{\tilde{t}}_{\varphi}$ can qualitatively be seen as such evidence, it is difficult to identify a more quantitative correspondence. The nondiagonal component has a nontrivial dependence on $\tilde{r}$ even when expressed in the normalised basis, and we have verified that the same holds also in a Lorentz-orthonormal basis in which the spacelike unit vector in span$\{\partial_{\tilde{t}}, \partial_\varphi\}$ is proportional to $2\pi\partial_\varphi + \lambda_-\tilde{\eta}$.

3 Black holes with flat boundaries

In this section we investigate spacetimes obtained from the Schwarzschild-AdS family in $d \geq 4$ dimensions by replacing the round S^{d-2} by a flat space [45, 46, 47, 48, 49]. The metric in the curvature coordinates $(x^\mu) = (t, r, \vec{x})$ reads

$$\mathrm{d}s^2 = -f\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f} + \frac{r^2}{\ell^2}\mathrm{d}\vec{x}^2 \quad , \tag{3.1a}$$

$$f(r) := \frac{r^2}{\ell^2} - \frac{2M}{r^{d-3}} \quad , \tag{3.1b}$$

where $\vec{x}^2 := \sum_{i=1}^{d-2}(x^i)^2$. The positive parameter ℓ is related to the cosmological constant by $\Lambda = -\frac{1}{2}(d-1)(d-2)\ell^{-2}$, and the positive parameter M is proportional to the ADM mass per unit coordinate volume in $\mathrm{d}\vec{x}^2$ [65, 66]. The value of r at the horizon, where f vanishes, is $r_h := \sqrt[d-1]{2M\ell^2}$. The surface gravity of the horizon, normalised to the Killing vector ∂_t, is $\kappa = \frac{1}{2}(d-1)r_h\ell^{-2}$, and the inverse Hawking temperature is $\beta_H := 2\pi/\kappa = \frac{4\pi}{(d-1)}\ell^2 r_h^{-1}$.

We take initially $\vec{x} \in \mathbb{R}^{d-2}$, so that the horizon has (spatial) topology $\mathbb{R}^{d-2}$. The parameter M has then coordinate-invariant meaning only in being positive, as its value can be rescaled by rescaling r, t and $\vec{x}$. Other horizon topologies will be discussed in subsections 3.3 and 3.4.

The metric on hypersurfaces of constant r satisfies

$$\mathrm{d}s^2\big|_{r=\mathrm{const}} \sim \frac{r^2}{\ell^2}\left(-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) \quad , \quad r \to \infty \quad , \tag{3.2}$$

and a convenient representative of the conformal equivalence class of metrics at the infinity is the $(d-1)$-dimensional Minkowski metric,

$$\mathrm{d}s^2_{\mathrm{CFT}} = -\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 \quad . \tag{3.3}$$

By AdS/CFT correspondence, one expects the bulk to induce on the boundary (3.3) a thermal state with inverse temperature β_H and energy expectation value proportional to M. The periodicity of t on the Euclidean-signature section of the bulk implies that the boundary two-point function evaluated in the geodesic approximation is periodic in t with period $i\beta_H$, but the boundary stress-energy requires an explicit computation. We now embark on this computation.

3.1 Bulk geodesics

It is convenient to analyse the bulk geodesics first on the Euclidean-signature section and at the end continue to Lorentz-signature. We write $t = -i\tau$ and regard τ in this subsection as real.

The geodesic equations read

$$\frac{\mathrm{d}\tau}{\mathrm{d}\sigma} = \frac{C}{f(r)} \quad , \tag{3.4a}$$

$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}\sigma} = \frac{\ell\vec{C}}{r^2} \quad , \tag{3.4b}$$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\sigma}\right)^2 = \frac{P(r)}{\ell^2 r^{d-1}} \quad , \tag{3.4c}$$

where σ is the proper distance, C and $\vec{C}$ are constants, and

$$P(r) := r^{d+1} - \left(C^2\ell^2 + \vec{C}^2\right)r^{d-1} - 2M\ell^2 r^2 + 2M\ell^2\vec{C}^2 \quad . \tag{3.5}$$

We are interested in geodesics that begin and end at $r = \infty$ and have a turning point at $r = r_{\min} > r_h$, which is the largest zero of $P(r)$. We truncate these geodesics at $r = r_{\max}$, where the cutoff $r_{\max}$ will shortly be taken to infinity. Denoting by $\Delta\tau$ and $\Delta\vec{x}$ the differences in respectively τ and $\vec{x}$ between the endpoints of the truncated geodesic and by L the length of the truncated geodesic, (3.4) gives

$$\Delta\tau = 2C\ell^3 \int_{r_{\min}}^{r_{\max}} \frac{r^{\frac{3d-7}{2}} \mathrm{d}r}{\left(r^{d-1} - 2M\ell^2\right)\sqrt{P(r)}} \quad , \tag{3.6a}$$

$$\Delta\vec{x} = 2\vec{C}\ell^2 \int_{r_{\min}}^{r_{\max}} \frac{r^{\frac{d-5}{2}} \mathrm{d}r}{\sqrt{P(r)}} \quad , \tag{3.6b}$$

$$L = 2\ell \int_{r_{\min}}^{r_{\max}} \frac{r^{\frac{d-1}{2}} \mathrm{d}r}{\sqrt{P(r)}} \quad . \tag{3.6c}$$

Consider now the limit $r_{\max} \to \infty$ with fixed C and $\vec{C}$. $\Delta\tau$ and $\Delta\vec{x}$ tend to the finite values

$$\Delta\tau = 2C\ell^3 \int_{r_{\min}}^{\infty} \frac{r^{\frac{3d-7}{2}} \mathrm{d}r}{\left(r^{d-1} - 2M\ell^2\right)\sqrt{P(r)}} \quad , \tag{3.7a}$$

$$\Delta\vec{x} = 2\vec{C}\ell^2 \int_{r_{\min}}^{\infty} \frac{r^{\frac{d-5}{2}} \mathrm{d}r}{\sqrt{P(r)}} \quad . \tag{3.7b}$$

L diverges, but subtracting from (3.6c) the integral of $2\ell(r^2 - r_{\min}^2)^{-1/2}$ and performing the elementary integration shows that $L_{\mathrm{ren}} := \lim_{r_{\max}\to\infty}\left[L - 2\ell\ln(r_{\max}/\ell)\right]$ is finite and given by

$$L_{\mathrm{ren}} = 2\ell\ln\left(\frac{2\ell}{r_{\min}}\right) + 2\ell\int_{r_{\min}}^{\infty}\left(\frac{r^{\frac{d-1}{2}}}{\sqrt{P(r)}} - \frac{1}{\sqrt{r^2 - r_{\min}^2}}\right)\mathrm{d}r \quad . \tag{3.7c}$$

Recalling that $r_{\min}$ is determined by C and $\vec{C}^2$, we see that the system (3.7) determines L_{ren} at least locally as a function of $\Delta\tau$ and $\Delta\vec{x}$. We adopt this L_{ren} as the renormalised geodesic length to be used in the boundary Green's function.

To evaluate the boundary Green's function to quadratic order in $\Delta\tau$ and $\Delta\vec{x}$, which is the order that determines the boundary stress-energy, it turns out sufficient to find the expansion of L_{ren} to the next-to-leading order in $\Delta\tau$ and $\Delta\vec{x}$. We show in appendix B that this expansion is

$$L_{\mathrm{ren}} \sim 2\ell\ln(D/\ell) + \frac{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}{\Gamma\left(\frac{d+2}{2}\right)} \times \frac{MD^{d-3}}{2^d\ell^{2d-5}}\left[(\Delta\vec{x})^2 - (d-2)(\Delta\tau)^2\right] \quad , \tag{3.8}$$

where $D := \sqrt{(\Delta\tau)^2 + (\Delta\vec{x})^2}$.

3.2 Boundary CFT

We define on the Euclidean-signature section of the conformal boundary (3.3) the Green's function

$$G_{\mathrm{CFT}} := \frac{\Gamma\left(\frac{d-3}{2}\right)}{4\pi^{(d-1)/2}\ell^{d-3}}\, \mathrm{e}^{-(d-3)L_{\mathrm{ren}}/(2\ell)} \quad . \tag{3.9}$$

From (3.8), the expansion of G_{CFT} to quadratic order in $\Delta\tau$ and $\Delta\vec{x}$ reads

$$\begin{aligned} G^{(2)}_{\mathrm{CFT}} = {} & \frac{\Gamma\left(\frac{d-3}{2}\right)}{4\pi^{(d-1)/2}} \times \\ & \times \left\{ \frac{1}{D^{d-3}} + \left[\frac{2\pi}{(d-1)\beta_H}\right]^{d-1} \frac{(d-3)\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)}{8\Gamma\left(\frac{d+2}{2}\right)} \left[(d-2)(\Delta\tau)^2 - (\Delta\vec{x})^2\right] \right\} \quad , \end{aligned} \tag{3.10}$$

where the superscript $^{(2)}$ indicates that only terms up to quadratic order in the coordinate separation have been kept. We have written M in terms of the inverse Hawking temperature β_H. The coefficient of L_{ren} and the overall factor in (3.9) have been chosen so that G_{CFT} has the short distance divergence of a free scalar field [59].

As $G^{(2)}_{\mathrm{CFT}}$ satisfies the Klein-Gordon equation, we can consistently view it as a free conformal scalar field Green's function and compute the stress-energy from the quadratic term. The classical expression for the stress-energy is now given by the first three terms in (2.22), with $g_{\mu\nu}$ replaced by the Minkowski metric (3.3). Continuing $G^{(2)}_{\mathrm{CFT}}$ to Lorentz-signature by $\tau = it$, the point-split calculation is straightforward and yields

$$\langle T_{\mu\nu}\rangle_{\mathrm{CFT}} = \frac{\left[\Gamma\left(\frac{d-1}{2}\right)\right]^2}{8\pi^{(d-2)/2}\Gamma\left(\frac{d+2}{2}\right)} \left[\frac{2\pi}{(d-1)\beta_H}\right]^{d-1} \times \mathrm{diag}(d-2, 1, \ldots, 1) \quad . \tag{3.11}$$

Note that $\langle T_{\mu\nu}\rangle_{\mathrm{CFT}}$ is traceless. Expressing β_H in terms of M shows that (3.11) is proportional to M. The boundary energy density is therefore proportional to the black hole ADM mass. This is the result one would have expected.

It is of interest to compare $\langle T_{\mu\nu}\rangle_{\mathrm{CFT}}$ (3.11) to the stress-energy $\langle T_{\mu\nu}\rangle_{\mathrm{free}}$ of a free conformal scalar field on Minkowski space in an ordinary thermal state with inverse temperature β_H. From appendix C, first term in equation (C.10), we see that the expressions agree up to a d-dependent multiplicative constant. We have verified by a combination of numerical methods at small d and an asymptotic expansion at large d that $\langle T_{00}\rangle_{\mathrm{CFT}}/\langle T_{00}\rangle_{\mathrm{free}}$ is less than 1 for all integers $d > 3$ (although it equals 1 at $d \approx 3.1475$ and approaches 1 as $d \to 3$) and decreases rapidly as d increases.

3.3 A periodic boundary dimension

In this subsection we make one of the spatial dimensions in the bulk metric (3.1) periodic by $\left(t, r, x^1, x^2, \ldots, x^{d-2}\right) \sim \left(t, r, x^1 + a, x^2, \ldots, x^{d-2}\right)$, where a is a positive parameter. The conformal boundary (3.3) becomes then Minkowski space with one periodic spatial

dimension. We assume the extension past the horizon to be of the usual Kruskal-type, reviewed in appendix D. This guarantees that the Lorentz-signature continuation of G_{CFT} (3.9) will receive no contribution from geodesics that cross the horizon.

The stress-energy of a free conformal scalar field in an ordinary thermal state on Minkowski space with one periodic dimension, computed in appendix C, suggests that in the limit of large a with fixed M $\langle T_{\mu\nu}\rangle_{\mathrm{CFT}}$ (3.11) should receive a correction proportional to

$$\frac{1}{\beta_H a^{d-2}} \times \mathrm{diag}(0, 3-d, 1, \ldots, 1) \quad . \tag{3.12}$$

We now show that this suggestion is not realised.

A first observation is that any new contribution to $\langle T_{\mu\nu}\rangle_{\mathrm{CFT}}$ must be suppressed in a exponentially, rather than as a power-law. In (3.7), the coincidence limit of the new geodesics occurs when $\Delta\tau = \Delta x^2 = \cdots = \Delta x^{d-2} = 0$ but $\Delta x^1 = na$, $n \in \mathbb{Z} \setminus \{0\}$. This implies $C = 0$, $\vec{C}^2 = r_{\min}^2$ and $P(r) = (r^2 - r_{\min}^2)\big(r^{d-1} - r_h^{d-1}\big)$. The limit of large $|n|a$ is the limit $r_{\min} \to r_h$, in which (3.7b) and (3.7c) give

$$\frac{|n|a}{\ell} \sim -\sqrt{\frac{2}{d-1}}\,\frac{\ell}{r_h}\ln\!\left(\frac{r_{\min}}{r_h} - 1\right) \quad , \tag{3.13a}$$

$$\frac{L_{\mathrm{ren}}}{\ell} \sim -\sqrt{\frac{2}{d-1}}\ln\!\left(\frac{r_{\min}}{r_h} - 1\right) \quad , \tag{3.13b}$$

and hence $L_{\mathrm{ren}} \sim (r_h/\ell)|n|a$. The dominant correction in $G^{(2)}_{\mathrm{CFT}}$ thus comes from geodesics close to the coincidence geodesics with $n = \pm 1$, and this contribution involves the suppression factor $\mathrm{e}^{-(d-3)L_{\mathrm{ren}}/(2\ell)} \sim \exp\!\big[-\tfrac{1}{2}(d-3) r_h a \ell^{-2}\big]$.

Computing the correction in $G^{(2)}_{\mathrm{CFT}}$ requires an expansion around the coincidence geodesics with $n = \pm 1$. What makes the calculation laborious is that the leading contribution from each geodesic is linear in the coordinate separation, and the linear terms only cancel on adding the contributions from $n = 1$ and $n = -1$. We have only performed this calculation for $d = 5$, in which case the integrals in (3.7) simplify by taking r^2 as the new variable.[2] Omitting here the steps, we find that the correction in $G^{(2)}_{\mathrm{CFT}}$ reads

$$\frac{\Gamma\big(\frac{d-3}{2}\big)}{4\pi^{(d-1)/2}} \times \frac{\big(1+\sqrt{2}\big)^2}{4} \exp\!\big(-r_h a \ell^{-2}\big) \frac{r_h^4}{\ell^8} \big(\delta x^1\big)^2 \quad , \tag{3.14}$$

where δx^1 denotes the separation in periodically identified x^1. The terms involving $\Delta\tau$ and Δx^i for $2 \le i \le d-2$ are subleading in a.

As (3.14) does not satisfy the Klein-Gordon equation, we cannot interpret it as a correction in a free conformal scalar field Green's function. We conclude that our AdS/CFT correspondence model cannot be used to compute the periodic correction to the boundary stress-energy at least for $d = 5$.

[2] The integrals can be evaluated in terms of elliptic integrals. We have however not been able to exploit this observation in the calculations.

3.4 Geon

Similarly to the $\mathbb{RP}^3$ extension of Schwarzschild [33, 34, 35, 62] and the $\mathbb{RP}^2$ extension of nonrotating BTZ [12], it is possible to continue the exterior metric (3.1) with periodic x^1 into inextendible spacetimes that do not contain a second exterior [67]. The interest of these unconventional extensions for our AdS/CFT correspondence model is that the Lorentz-signature continuation of G_{CFT} (3.9) may then receive additional contributions from geodesics that cross the horizon. In the $\mathbb{RP}^2$ case such geodesics are required to recover the appropriate Green's function on the boundary [14], and we observed a similar phenomenon in the HD-BTZ case in subsection 2.5. We now address this question with the metric (3.1).

We consider the spacetime obtained from the Kruskal-type extension of appendix D by the identification $\left(U, V, x^1, x^2, \ldots, x^{d-2}\right) \sim \left(V, U, x^1 + \frac{1}{2}a, x^2, \ldots, x^{d-2}\right)$. The global structure differs from that of the $\mathbb{RP}^2$ geon [12] by the $d-3$ dimensions that are inert in the identification, and also by the fact that the singularities in the conformal diagram cave inward compared with the infinities (cf. [29]). The exterior is as in subsection 3.3, but there are now horizon-crossing geodesics that begin and end at the conformal boundary.

Let $t = 0$ be the distinguished constant t hypersurface in the exterior [12, 62]. By the results in subsection 2.5, one expects the contribution from the horizon-crossing geodesics to be exponentially suppressed as $|t| \to \infty$ with fixed a. However, at sufficiently small $|t|$, the horizon-crossing geodesics dominate those considered in subsection 3.3. To see this, consider the horizon-crossing coincidence limit geodesics at $t = 0$. These geodesics belong to the Euclidean-signature section at $\tau = 0$ and are obtained from (3.7) with $C = 0$, $r_{\min} = r_h$, $\vec{C}^2 < r_h^2$, $P(r) = \left(r^2 - \vec{C}^2\right)\left(r^{d-1} - r_h^{d-1}\right)$, $\Delta\tau = \Delta x^2 = \cdots = \Delta x^{d-2} = 0$ and $\Delta x^1 = \left(\frac{1}{2} + n\right)a$, $n \in \mathbb{Z}$. In parallel with (3.13), we now find $L_{\mathrm{ren}} \sim (r_h/\ell)\left|\frac{1}{2} + n\right|a$. The dominant correction in $G^{(2)}_{\mathrm{CFT}}$ thus comes from geodesics close to the coincidence geodesics with $n = 0$ and $n = -1$ and involves the suppression factor $\mathrm{e}^{-(d-3)L_{\mathrm{ren}}/(2\ell)} \sim \exp\left[-\frac{1}{4}(d-3)r_h a \ell^{-2}\right]$, which goes to zero less rapidly than the factor $\exp\left[-\frac{1}{2}(d-3)r_h a\ell^{-2}\right]$ found in subsection 3.3.

We have computed the leading correction in $G^{(2)}_{\mathrm{CFT}}$ only for $d = 5$, and then only at $t = 0$. Written on the Lorentz-signature section in the notation of (3.14), we find that this correction is

$$\frac{\Gamma\left(\frac{d-3}{2}\right)}{4\pi^{(d-1)/2}} \times \frac{\left(1+\sqrt{2}\right)^2}{4} \exp\left(-\tfrac{1}{2} r_h a \ell^{-2}\right) \frac{r_h^4}{\ell^8} \left[\left(\delta x^1\right)^2 + 3\sqrt{2}(\Delta t)^2\right] \quad . \tag{3.15}$$

As (3.15) does not satisfy the Klein-Gordon equation, we cannot interpret it as a correction in a free conformal scalar field Green's function. Although the horizon-crossing geodesics give the leading a-dependent correction in the Green's function, we therefore cannot use them to compute a correction to the boundary stress-energy within our AdS/CFT model at least for $d = 5$.

4 Discussion

In this paper we have analysed AdS/CFT correspondence for two families of Einstein black holes in $d \geq 4$ dimensions, modelling the boundary CFT by a free conformal scalar field and evaluating the boundary two-point function semiclassically from bulk geodesics. For the HD-BTZ hole, which is locally AdS_d and generalises the nonrotating BTZ hole to $d \geq 4$, the model was fully self-consistent. The boundary state was the Euclidean vacuum induced from the global vacuum on the conformal boundary of AdS_d, which is thermal in the sense appropriate for the dS_{d-2} factor in the boundary metric. The boundary stress-energy suggested a novel definition for the mass of the HD-BTZ hole by AdS/CFT: In the absence of an ADM mass, the interest in this definition remains to be seen. We also analysed briefly a $\mathbb{Z}_2$ quotient and a generalisation involving rotation.

For the generalised $d \geq 4$ Schwarzschild-AdS hole with a flat $\mathbb{R}^{d-2}$ horizon, the model was self-consistent at the level of the boundary stress-energy, and the stress-energy had the thermal form in a temperature that agreed with the hole Hawking temperature up to a d-dependent numerical factor. In particular, the energy density was proportional to the hole ADM mass. However, the model could not consistently accommodate corrections from a periodic horizon dimension in the limit of large period with fixed mass for $d = 5$. Similarly, the model could not consistently accommodate corrections from geodesics that cross the horizons in a single-exterior version of this $d = 5$ hole, obtained as a $\mathbb{Z}_2$ quotient. We suspect these inconsistencies to be present for all $d \geq 4$.

The stress-energy tensor on the boundary of the HD-BTZ hole received contributions from bulk geodesics that pass through the near-horizon region, and for the $\mathbb{Z}_2$ quotient there were additional contributions from geodesics that pass inside the horizon. These 'long' geodesics arise from the construction of the hole as a quotient of AdS_d and have no counterpart in AdS_d itself. It follows that these contributions represent an effect that is not present in a boundary stress-energy tensor calculated from the near-infinity metric by quasilocal techniques with counterterm subtraction [68, 69, 70]: The quasilocal stress-energy tensor is insensitive to quotients that preserve the conformal rescaling near the infinity and cannot thus depend on the parameter λ_+ that determines the size of the hole. For the explicit computation of the quasilocal stress-energy for $d = 5$, see [71]. The parameter λ_+ would affect integration of the quasilocal stress-energy, and this is how the quasilocal stress-energy on the boundary of the BTZ hole reproduces the correct mass and angular momentum as conserved charges [68], but the absence of a global timelike Killing vector prevents a direct lift of this BTZ result to the HD-BTZ boundary. For the generalised Schwarzschild-AdS hole, our stress-energy result for nonperiodic horizon agrees with the quasilocal stress-energy [72].

Why did a period on the horizon of the generalised Schwarzschild-AdS hole make the model inconsistent? One might suspect the reason to have been an inappropriate matching of the boundary and the bulk.[3] In the semiclassical evaluation of the boundary two-point function, the bulk geometry should be regarded as a saddle point in the

[3]We thank Rob Myers for raising this possibility.

gravitational path integral under boundary conditions set by the boundary geometry. When the Euclidean-signature boundary has two periodic dimensions, there are a priori two families of saddle points, differing in the choice of the boundary circumference that is matched to the bulk Euclidean time, and for the dominant saddle point this circumference is the smaller one [73, 74, 75].[4] As we considered a limit of large spatial period with fixed hole mass, our bulk was in fact the dominant saddle point, so this cannot have caused the problem.

The reason for the inconsistency may be just that a free conformal scalar field is not a good boundary CFT model when bulk regions where the local geometry differs substantially from AdS_d become important: We see from (3.13) that in the limit of large period, the troublesome geodesics pass arbitrarily close to the horizon. For the ordinary Schwarzschild-AdS hole, it was indeed argued in [29] that a better boundary CFT model is a high conformal weight scalar field.[5] By adjusting the parameter μ in (2.17), our geodesic length analyses yield small separation expansions for arbitrary conformal weight Green's functions on the boundary, but gaining useful information from such an expansion is more difficult when the field is not free. For a discussion of the conformal anomaly, see [79].

It would be possible to evaluate boundary Wilson loops semiclassically for our generalised Schwarzschild-AdS holes and analyse the corrections that arise from the choice of the bulk topology. Our Green's function results suggest, however, that one should first develop a better understanding of what might constitute a reasonable boundary CFT model for these bulks.

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A Asymptotics for subsection 2.5

In this appendix we prove a lemma needed for the large $|T|$ behaviour (2.33).

[4] For similar issues with a finite distance boundary, see [76, 77, 78].

[5] We thank Veronika Hubeny for this observation.

Lemma A.1 *Let $\lambda_+ > 0$ and $p > 0$. For $\alpha > 0$, let*

$$F(\alpha) := \sum_{k=0}^{\infty} \frac{1}{\bigl\{\cosh\bigl[(2k+1)\pi\lambda_+\bigr] + \alpha\bigr\}^p} \quad , \tag{A.1a}$$

$$G(\alpha) := \sum_{k=0}^{\infty} \frac{1}{\bigl[\frac{1}{2}\mathrm{e}^{(2k+1)\pi\lambda_+} + \alpha\bigr]^p} \quad . \tag{A.1b}$$

Then $G(\alpha) = \mathcal{O}\left(\frac{\ln\alpha}{\alpha^p}\right)$ and $F(\alpha) = G(\alpha) + \mathcal{O}\bigl(\alpha^{-(p+1)}\bigr)$ as $\alpha \to \infty$.

Proof. Consider first G. We write $G(\alpha) = \bigl(2\mathrm{e}^{-\pi\lambda_+}\bigr)^p S(2\mathrm{e}^{-\pi\lambda_+}\alpha)$, where

$$S(x) := \sum_{k=0}^{\infty} \frac{1}{\bigl(\mathrm{e}^{2\pi\lambda_+ k} + x\bigr)^p} \quad . \tag{A.2}$$

Estimating the sum in (A.2) by the integral gives the sandwich inequality $I(x) < S(x) < I(x) + (1+x)^{-p}$, where

$$\begin{aligned}
I(x) &:= \int_0^\infty \frac{\mathrm{d}t}{(\mathrm{e}^{2\pi\lambda_+ t} + x)^p} \\
&= \frac{1}{2\pi\lambda_+ x^p} \int_{1/x}^\infty \frac{\mathrm{d}z}{z(z+1)^p} \\
&= \frac{1}{2\pi\lambda_+ x^p} \left\{ \int_{1/x}^\infty \frac{\mathrm{d}z}{z(z+1)} + \int_0^\infty \frac{\mathrm{d}z}{z}\left[\frac{1}{(z+1)^p} - \frac{1}{z+1}\right] \right. \\
&\qquad\qquad \left. - \int_0^{1/x} \frac{\mathrm{d}z}{z}\left[\frac{1}{(z+1)^p} - \frac{1}{z+1}\right] \right\} \\
&= \frac{1}{2\pi\lambda_+ x^p} \left\{ \ln x + \int_0^\infty \frac{\mathrm{d}z}{z}\left[\frac{1}{(z+1)^p} - \frac{1}{z+1}\right] + \mathcal{O}(x^{-1}) \right\} \quad .
\end{aligned} \tag{A.3}$$

In (A.3) we have first changed variables by $\mathrm{e}^{2\pi\lambda_+ t} = xz$ and then rearranged the integral in a form whose large x expansion is elementary. The sandwich inequality and (A.3) imply $S(x) = \mathcal{O}\left(\frac{\ln x}{x^p}\right)$, and hence $G(\alpha) = \mathcal{O}\left(\frac{\ln\alpha}{\alpha^p}\right)$.

Consider then the difference of F and G. From (A.1) we find

$$\alpha^{p+1}\bigl[G(\alpha) - F(\alpha)\bigr] = \sum_{k=0}^{\infty} \alpha^{p+1} \left(\frac{1}{\bigl[\frac{1}{2}\mathrm{e}^{(2k+1)\pi\lambda_+} + \alpha\bigr]^p} - \frac{1}{\bigl\{\cosh\bigl[(2k+1)\pi\lambda_+\bigr] + \alpha\bigr\}^p} \right) \quad , \tag{A.4}$$

where the sum rearrangement is allowed by absolute convergence. An elementary analysis shows that the kth term on the right-hand side of (A.4) is positive and bounded above by $\frac{1}{2}p\mathrm{e}^{-(2k+1)\pi\lambda_+}$ and tends to $\frac{1}{2}p\mathrm{e}^{-(2k+1)\pi\lambda_+}$ as $\alpha \to \infty$. It follows by dominated

convergence that we can take the limit $\alpha \to \infty$ in (A.4) termwise, and summing the geometric series gives

$$\alpha^{p+1}\big[G(\alpha) - F(\alpha)\big] \xrightarrow[\alpha\to\infty]{} \frac{p}{4\cosh(\pi\lambda_+)} \quad . \tag{A.5}$$

Hence $F(\alpha) = G(\alpha) + \mathcal{O}\big(\alpha^{-(p+1)}\big)$. ■

B Expansion of $L_{\rm ren}$

In this appendix we solve the system (3.7) for $L_{\rm ren}$ to next-to-leading order in the limit of small $\Delta\tau$ and $\Delta\vec{x}$ with fixed $|\Delta\vec{x}|/|\Delta\tau|$.

We specify the geodesic in (3.7) by $\vec{C}$ and $r_{\rm min}$. From (3.5), C is then given by

$$C\ell = \sqrt{\big(r_{\rm min}^2 - \vec{C}^2\big)\left[1 - (r_h/r_{\rm min})^{d-1}\right]} \quad , \tag{B.1}$$

and $P(r)$ factorises as

$$P(r) = (r - r_{\rm min})(r^d + a_{d-1}r^{d-1} + \cdots + a_0) \quad , \tag{B.2a}$$

where

$$\begin{aligned}
a_{d-1} &= r_{\rm min} \quad , \\
a_{d-i} &= \frac{r_h^{d-1}}{r_{\rm min}^{d-i-1}}\left(1 - \frac{\vec{C}^2}{r_{\rm min}^2}\right) \quad , \quad 2 \le i \le d-2 \quad , \\
a_1 &= -\frac{r_h^{d-1}\vec{C}^2}{r_{\rm min}^2} \quad , \\
a_0 &= -\frac{r_h^{d-1}\vec{C}^2}{r_{\rm min}} \quad .
\end{aligned} \tag{B.2b}$$

We first expand (3.7a) and (3.7b) to next-to-leading order at large $r_{\rm min}$, keeping $\vec{C}/r_{\rm min}$ constant. From (3.7b), we obtain

$$\begin{aligned}
\Delta\vec{x} &= 2\vec{C}\ell^2 \int_{r_{\rm min}}^{\infty} \frac{r^{\frac{d-5}{2}}\mathrm{d}r}{\sqrt{(r - r_{\rm min})(a_0 + \cdots + r^d)}} \\
&= 2\vec{C}\ell^2 \int_{r_{\rm min}}^{\infty} \frac{\mathrm{d}r}{r^2\sqrt{r^2 - r_{\rm min}^2}\sqrt{1 + \frac{a_{d-2}}{r(r+r_{\rm min})} + \cdots + \frac{a_0}{r^{d-1}(r+r_{\rm min})}}} \\
&\sim 2\vec{C}\ell^2 \left[\int_{r_{\rm min}}^{\infty} \frac{\mathrm{d}r}{r^2\sqrt{r^2 - r_{\rm min}^2}} - \sum_{i=0}^{d-2} \frac{a_i}{2} \int_{r_{\rm min}}^{\infty} \frac{\mathrm{d}r}{r^{d+1-i}(r + r_{\rm min})\sqrt{r^2 - r_{\rm min}^2}}\right] \quad .
\end{aligned} \tag{B.3}$$

Note that all the terms under the sum in (B.3) contribute to the next-to-leading term, which is suppressed compared to the leading term by the factor $(r_h/r_{\rm min})^{d-1}$. The dropped terms are suppressed compared with the leading term by $(r_h/r_{\rm min})^{2(d-1)}$. A similar treatment of (3.7a) yields

$$\Delta\tau \sim 2C\ell^3 \int_{r_{\rm min}}^{\infty} \frac{\left[1 + \frac{r_h^{d-1}}{r^{d-1}} - \sum_{i=0}^{d-2} \frac{a_i}{2r^{d-1-i}(r+r_{\rm min})}\right] \mathrm{d}r}{r^2\sqrt{r^2 - r_{\rm min}^2}} \quad , \tag{B.4}$$

where C can be replaced by its next-to-leading order expansion

$$C\ell \sim \sqrt{r_{\rm min}^2 - \vec{C}^2}\left(1 - \frac{r_h^{d-1}}{2r_{\rm min}^{d-1}}\right) \quad . \tag{B.5}$$

The integrals in (B.3) and (B.4) can be evaluated by the identities, easily proved by induction,

$$\int_{r_{\rm min}}^{\infty} \frac{\mathrm{d}r}{r^{m+1}\sqrt{r^2 - r_{\rm min}^2}} = \frac{1}{r_{\rm min}^{m+1}} \frac{(m-1)!!}{m!!} \left(\frac{\pi}{2}\right)^{\frac{1}{2}[1+(-1)^m]} \quad , \tag{B.6a}$$

$$\int_{r_{\rm min}}^{\infty} \frac{\mathrm{d}r}{r^m(r+r_{\rm min})\sqrt{r^2 - r_{\rm min}^2}} = \frac{1}{r_{\rm min}^{m+1}} \sum_{i=0}^{m} (-1)^i \frac{(m-i-2)!!}{(m-i-1)!!} \left(\frac{\pi}{2}\right)^{\frac{1}{2}[1-(-1)^{m-i}]} \quad , \tag{B.6b}$$

where $m \geq 0$ and the double factorial of a negative number is understood to be unity. Collecting, we obtain

$$\begin{aligned}\Delta\tau \sim{}& \frac{2\ell^2}{r_{\rm min}^2}\sqrt{r_{\rm min}^2 - \vec{C}^2} \\ &\times \left[1 + \frac{\alpha_d}{2}\left(\frac{\vec{C}^2}{r_{\rm min}^2}\right)\frac{r_h^{d-1}}{r_{\rm min}^{d-1}} + \left(\alpha_d - \tfrac{1}{2}\right)\frac{r_h^{d-1}}{r_{\rm min}^{d-1}} - \frac{\beta_d}{2}\left(1 - \frac{\vec{C}^2}{r_{\rm min}^2}\right)\frac{r_h^{d-1}}{r_{\rm min}^{d-1}}\right] \quad ,\end{aligned} \tag{B.7a}$$

$$\Delta\vec{x} \sim \frac{2\vec{C}\ell^2}{r_{\rm min}^2}\left[1 + \frac{\alpha_d}{2}\left(\frac{\vec{C}^2}{r_{\rm min}^2}\right)\frac{r_h^{d-1}}{r_{\rm min}^{d-1}} - \frac{\beta_d}{2}\left(1 - \frac{\vec{C}^2}{r_{\rm min}^2}\right)\frac{r_h^{d-1}}{r_{\rm min}^{d-1}}\right] \quad , \tag{B.7b}$$

where

$$\alpha_d := \frac{(d-1)!!}{d!!}\left(\frac{\pi}{2}\right)^{\frac{1}{2}[1+(-1)^d]} \quad , \tag{B.8a}$$

$$\beta_d := [1 + (-1)^d]\left(\frac{3\pi}{8} - 1\right) + \sum_{k=1}^{[\frac{d-3}{2}]} \frac{(d-2k-1)!!}{(d-2k)!!}\left(\frac{\pi}{2}\right)^{\frac{1}{2}[1+(-1)^d]} \quad . \tag{B.8b}$$

The square bracket in the sum limit in (B.8b) denotes integer part.

Next, we invert (B.7) for $\vec{C}$ and $r_{\min}$ to the next-to-leading order in the limit of small $\Delta\tau$ and $\Delta\vec{x}$ with fixed $|\Delta\vec{x}|/|\Delta\tau|$. The result is

$$r_{\min} \sim \frac{2\ell^2}{D}\left\{1 + \frac{MD^{d-3}}{2^{d-1}\ell^{2d-4}}\left[\alpha_d(\Delta\vec{x})^2 + \big(2\alpha_d - \beta_d - 1\big)(\Delta\tau)^2\right]\right\} \quad , \tag{B.9a}$$

$$\frac{\vec{C}^2}{r_{\min}^2} \sim \frac{(\Delta\vec{x})^2}{D^2}\left[1 + \frac{MD^{d-3}}{2^{d-2}\ell^{2d-4}}\big(2\alpha_d - 1\big)(\Delta\tau)^2\right] \quad , \tag{B.9b}$$

where $D := \sqrt{(\Delta\tau)^2 + (\Delta\vec{x})^2}$. Comparison of (B.9) and our large $r_{\min}$ expansion of (3.7a) and (3.7b) shows that this large $r_{\min}$ expansion kept all terms that contribute to the next-to-leading order in $\Delta\tau$ and $\Delta\vec{x}$. Hence (B.9) solves (3.7a) and (3.7b) to the order shown.

Finally, the integral in (3.7c) can be expanded by similar techniques. We omit the details. Using (B.9) and writing the remaining double factorial in terms of the gamma-function gives (3.8).

C Finite temperature stress-energy with one periodic dimension

In this appendix we find the leading finite size correction to the finite temperature stress-energy tensor of a free conformal scalar field on Minkowski spacetime with one periodic spacelike dimension. Despite the interest of finite size effects in finite temperature field theory (see for example [80]), we have found this correction in the literature only in dimension two ([59], Section 4.2). In four dimensions, the electromagnetic finite temperature Casimir effect between two perfectly conducting planes is closely similar [81].

Following the conventions of the main text, the Minkowski dimension is $d-1$ with $d > 3$. The coordinates on the Euclidean-signature section are $(\tau, x, \vec{y})$ with $\vec{y} \in \mathbb{R}^{d-3}$ and $(\tau, x, \vec{y}) \sim (\tau, x + a, \vec{y})$. The inverse temperature is β.

Let

$$\widetilde{G}_0(\tau, x, \vec{y}) := \frac{1}{\big(\tau^2 + x^2 + \vec{y}^2\big)^{(d-3)/2}} \quad . \tag{C.1}$$

By the method of images, the Green's function is

$$G = \frac{\Gamma\big(\frac{d-3}{2}\big)}{4\pi^{(d-1)/2}}\widetilde{G} \quad , \tag{C.2}$$

where

$$\begin{aligned}\widetilde{G}(\tau, x, \vec{y}) &= \widetilde{G}_0(\tau, x, \vec{y}) \\ &\quad + \sum_{m,n}{}' \left[\tfrac{1}{2}\widetilde{G}_0(\tau + m\beta, x + na, \vec{y}) + \tfrac{1}{2}\widetilde{G}_0(\tau - m\beta, x - na, \vec{y}) - \widetilde{G}_0(m\beta, na, \vec{0})\right]\end{aligned} \tag{C.3}$$

and the primed sum is over $(m,n) \in \mathbb{Z}^2 \setminus \{(0,0)\}$. The inclusion of the constant terms and the $(m,n) \leftrightarrow (-m,-n)$ pairing make the sum convergent in absolute value for all $d > 3$.

Lemma C.1 *Let $p > 1$. For $u > 0$, let*

$$g_p(u) := \sum_{m,n=1}^{\infty} \frac{1}{(m^2 + n^2u^2)^p} \quad . \tag{C.4}$$

Then

$$g_p(u) = \frac{\sqrt{\pi}\,\Gamma\bigl(p - \frac{1}{2}\bigr)\zeta(2p-1)}{2\Gamma(p)\,u^{2p-1}} + \mathcal{O}\bigl(u^{-2p}\bigr) \tag{C.5}$$

as $u \to \infty$, where ζ is the Riemann zeta-function, and the expansion (C.5) can be differentiated in u.

Proof. An integral estimate similar to that in the proof of Lemma A.1 yields

$$\sum_{m=1}^{\infty} \frac{1}{(m^2+v^2)^p} = \frac{\sqrt{\pi}\,\Gamma\bigl(p-\frac{1}{2}\bigr)}{2\Gamma(p)\,v^{2p-1}} + \mathcal{O}\bigl(v^{-2p}\bigr) \tag{C.6}$$

as $v \to \infty$, where the coefficient in the leading term arises as the integral $\int_0^\infty (1+y^2)^{-p}\mathrm{d}y$ [53]. Setting in (C.6) $v = nu$ and summing over n, (C.5) follows using the infinite sum representation of the zeta-function [53]. To justify differentiation of (C.5), multiply (C.6) by powers of n^2, set $v = nu$ and sum over n, and compare with derivatives of (C.4). ■

We split $\widetilde{G}$ as

$$\widetilde{G} = \widetilde{G}_0 + \widetilde{G}_1 + \widetilde{G}_2 + \widetilde{G}_3 \quad , \tag{C.7}$$

where $\widetilde{G}_1$ consists of the $n = 0 \neq m$ terms, $\widetilde{G}_2$ consists of the $m = 0 \neq n$ terms and $\widetilde{G}_3$ consists of the $m \neq 0 \neq n$ terms. In $\widetilde{G}_1$, $\widetilde{G}_2$ and $\widetilde{G}_3$ we expand the summands to quadratic order in τ, x and $\vec{y}$. Interchanging the sum and the expansion, justified by convergence arguments that we omit here, we find

$$\widetilde{G}_1^{(2)} = \frac{(d-3)\zeta(d-1)}{\beta^{d-1}}\left[(d-2)\tau^2 - x^2 - \vec{y}^2\right] \quad , \tag{C.8a}$$

$$\widetilde{G}_2^{(2)} = \frac{(d-3)\zeta(d-1)}{a^{d-1}}\left[(d-2)x^2 - \tau^2 - \vec{y}^2\right] \quad , \tag{C.8b}$$

$$\begin{aligned}
\widetilde{G}_3^{(2)} &= 2(d-3)\sum_{m,n=1}^{\infty}\left[\frac{(d-2)\tau^2 - x^2 - \vec{y}^2}{\bigl(m^2\beta^2 + n^2a^2\bigr)^{(d-1)/2}} + \frac{(d-1)n^2a^2\bigl(x^2-\tau^2\bigr)}{\bigl(m^2\beta^2+n^2a^2\bigr)^{(d+1)/2}}\right] \\
&= 2(d-3)\beta^{-(d-1)}\Bigl\{\left[(d-2)\tau^2 - x^2 - \vec{y}^2\right]g_{(d-1)/2}\bigl(a\beta^{-1}\bigr) \\
&\qquad + \bigl(\tau^2 - x^2\bigr)a\beta^{-1}g'_{(d-1)/2}\bigl(a\beta^{-1}\bigr)\Bigr\} \quad ,
\end{aligned} \tag{C.8c}$$

where the superscript $^{(2)}$ indicates that only terms up to quadratic order have been kept and the prime on $g_{(d-1)/2}$ denotes derivative with respect to the argument.

In the limit of large a with fixed β, (C.5) and (C.8c) imply

$$\widetilde{G}_3^{(2)} = \frac{2\sqrt{\pi}\,\Gamma\bigl(\frac{d-2}{2}\bigr)\zeta(d-2)}{\Gamma\bigl(\frac{d-3}{2}\bigr)\beta a^{d-2}}\left[(d-3)x^2 - \vec{y}^2\right] + \mathcal{O}\bigl(a^{-(d-1)}\bigr) \quad . \tag{C.9}$$

Continuing G to Lorentz-signature, standard point-splitting methods [59] give

$$\begin{aligned}\langle T_{\mu\nu}\rangle = {} & \frac{\Gamma\bigl(\frac{d-1}{2}\bigr)\zeta(d-1)}{\pi^{(d-1)/2}\beta^{d-1}} \times \mathrm{diag}(d-2,1,\ldots,1) \\ & + \frac{\Gamma\bigl(\frac{d-2}{2}\bigr)\zeta(d-2)}{\pi^{(d-2)/2}\beta a^{d-2}} \times \mathrm{diag}(0,3-d,1,\ldots,1) \\ & + \mathcal{O}\bigl(a^{-(d-1)}\bigr) \quad ,\end{aligned} \tag{C.10}$$

where the first term is the usual Minkowski value, coming from $\widetilde{G}_1^{(2)}$, and the second term is the leading correction, coming from $\widetilde{G}_3^{(2)}$. Note that this correction dominates the zero-temperature vacuum polarisation term that comes from $\widetilde{G}_2^{(2)}$ and is proportional to $a^{-(d-1)}$.

D Kruskal extension

In this appendix we present the Kruskal-type extension of the metric (3.1). For notational convenience we write $d = n+1$, $n \geq 3$.

Starting in the exterior region $r > r_h$, we define the tortoise coordinate r_* by

$$\begin{aligned} r_* &:= -\int_r^\infty \frac{\mathrm{d}\tilde{r}}{f(\tilde{r})} \\ &= \frac{\ell^2}{n r_h}\left[\ln\left(\frac{r-r_h}{r+r_h}\right) - h(r_h/r)\right] \quad ,\end{aligned} \tag{D.1}$$

where

$$h(s) := \int_0^s \left(\frac{n}{1-z^n} - \frac{2}{1-z^2}\right)\mathrm{d}z \quad , \tag{D.2}$$

and the Kruskal coordinates $(U, V, \vec{x})$ by

$$U := -\exp\left[-\frac{n r_h (t - r_*)}{2\ell^2}\right] \quad , \tag{D.3a}$$

$$V := \exp\left[\frac{n r_h (t + r_*)}{2\ell^2}\right] \quad . \tag{D.3b}$$

The metric reads

$$\mathrm{d}s^2 = -\frac{4\ell^2 (r+r_h)\bigl(r^n - r_h^n\bigr)}{n^2 r_h^2 r^{n-2}(r-r_h)} \mathrm{e}^{h(r_h/r)}\,\mathrm{d}U\mathrm{d}V + \frac{r^2}{\ell^2}\,\mathrm{d}\vec{x}^2 \quad , \tag{D.4a}$$

where r is determined as a function of U and V by

$$-UV = \left(\frac{r - r_h}{r + r_h}\right) \mathrm{e}^{-h(r_h/r)} \quad . \tag{D.4b}$$

The function $h(s)$ (D.2) is nonsingular for $0 \leq s < \infty$, $h(0) = 0$ and $\lim_{s\to\infty} h(s) = \pi \cot(\pi/n)$, using 3.241.3 in [53].[6] It follows by standard considerations that (D.4) is a global chart with $-1 < UV < \mathrm{e}^{-\pi \cot(\pi/n)}$.

As the infinities are at $UV \to -1$ and the singularities are at $UV \to \mathrm{e}^{-\pi \cot(\pi/n)} < 1$, the singularities in the conformal diagram cave inward relative to the infinities [82]. In the AdS/CFT context, this gives rise to phenomena analysed for Schwarzschild-AdS holes in [29].

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